Non-equilibrium phonon dynamics in trapped ion systems

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We propose a concrete experiment to probe the non-equilibrium local dynamics of the one-dimensional Bose-Hubbard model using a trapped ion system consisting of a linear chain of few Ba^+ ions prepared in a state of transverse motional mode which corresponds to a fixed number of phonons per ion. These phonons are well-known to be described by an effective Bose-Hubbard model. We propose a protocol which leads to a sudden local sign reversal of the on-site interaction strength of this Hubbard model at one of the sites and demonstrate that the subsequent non-equilibrium dynamics of the model can be experimentally probed by measuring the time-dependent phonon number in a specific motional state of the Ba^+ ions. We back our experimental proposal with exact numerical calculation of the dynamics of a Bose-Hubbard model subsequent to a local quench.

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The possibility of the use of systems of ultracold atoms and trapped ions to emulate theoretical models of strongly correlated condensed matter systems has been intensely investigated in recent years [1–4]. One of the first examples in this regard constituted the realization of the Bose-Hubbard model in optical lattice systems and characterization of the superfluid-insulator transition of the model via measurement of momentum distribution of the bosons [1]. More recently, there has been several theoretical proposals of emulating various condensed matter models in trapped ion systems [5–8]. The main advantage of the trapped ion emulators over their ultracold atom counterparts is that they have relatively large separation between individual ions compared to that between lattice sites of cold atoms in an optical lattice [4]. This allows optical measurement of the properties of the vibrational states of individual ions and thus enables one to address the local properties of the underlying strongly correlated model emulated by these systems. Also, these systems provide unique opportunity to study the Bose-Hubbard model in presence of an attractive on-site interaction.

In recent years, it has also been realized that ultracold atomic systems, which are probably the best examples of experimental realization of closed quantum systems, provide an unique opportunity for studying the non-equilibrium dynamics of their constituent atoms near a quantum critical point [9, 10]. This has led to a plethora of theoretical studies of such dynamics [11]. However, in all of these experiments, it has not been possible to achieve protocols which takes the system out of equilibrium by a *local* change of system parameters. This is mainly due to several experimental difficulties associated with addressing individual atoms in an optical lattice setup. Consequently, in most of the theoretical studies of non-equilibrium dynamics undertaken on these systems so far [12–14], the effect of a global time-dependent ramp of a system parameter has been studied in details; no attempts have been made to study the response of the system to, for example, a local ramp. Trapped ion emulators provide us an ideal test bed for studying such dynamics.

In this work, we demonstrate, via proposal of a concrete experiment, that a system of one-dimensional (1D) trapped ions provide a requisite experimental setup to study the non-equilibrium dynamics of a finite-sized 1D Bose-Hubbard model subsequent to a local quench. The key differences between our proposal and the experimental or theoretical studies carried out on cold-atom system earlier are two fold. First, our proposed setup provides an implementation of a local quench which changes the on-site interaction of the emulated model at a given ion site (or equivalently at a lattice site for phonons). Second, it allows optical measurement of local physical quantities such as the time variation of projection of the phonon (boson) number into a motional state at a given site and hence provide us with direct information regarding *local* phonon (boson) number variation during non-equilibrium dynamics subsequent to the quench. We provide a detailed description of the proposed experimental setup and chart out the parameter space in which such an experiment can be carried out without violating the phonon number conserving approximation. We also back our experimental proposal by an exact numerical study of the non-equilibrium dynamics of the finite-size Bose-Hubbard model emulated by the trapped ions. We compute the time evolution of the local boson (phonon) number density n_i subsequent to a sudden local sign change of the interaction ($U_i \to -U_i$), show that the variation of n_i can be observed experimentally by measuring the time dependence of the motional ground state phonon occupation number at that site (n_{i0}), and demonstrate that the amplitude of the quantum oscillations of n_{i0} is maximal when J/U_i lies in the critical (crossover for finite-size system) region. We note that dynamical properties of the Bose-Hubbard model emulated by an ionic system subsequent to a local quench has not been stud-

ied either experimentally or theoretically so far; our proposed experiment and the supporting theoretical analysis therefore constitute a significant extension of our understanding of local non-equilibrium dynamics of correlated boson systems emulated by trapped ions.

The experimental setup which we propose in the present work constitutes a linear chain of few Barium ions as shown schematically in middle and right panels of Fig. 1. The relevant energy level diagram of a singly charged barium ion is shown in the left panel of Fig. 1. We note here that most of our observations hold regardless of the nature of the used ions as long as they can undergo side-band cooling. For preparing the experimental setup, we trap these ions in a linear trap operated at 15 MHz radio-frequency and with the trap stability parameter $q \sim 0.42$ used for the radial confinement. This will generate a pseudo-potential in the radial plane corresponding to ion oscillation frequency $\omega_x \simeq 2.25$ MHz. The confinement in the axial direction is only by DC voltages applied to the endcap electrodes. This can be made shallow so that the axial frequency is ~ 180 kHz and the mean distance between the ions can be made of the order to 20 μm [15]. These parameters will lead to a tunneling strength of $J \simeq 0.55$ kHz. This in turn leads to $\beta_x = 2J/\omega_x$, the ratio between the coulomb interaction and the trapping potential, to be $\simeq 5 \times 10^{-4}$. In a linear trap like the one presented here, the inter ionic distance in the Coulomb crystal varies along the chain; however, for the present purpose this has been neglected as $\beta_x \ll 1$.

Next, we consider the on-site interaction for the phonons. For this, we note that for Doppler cooling of the ions, we use a diode laser at 493 nm. In addition, the side band cooling can be performed by driving the red-sideband near the S-D transition frequency at 2051 nm. In order to generate the on-site interaction, we use an off-resonant standing wave laser at 300 nm which results in a shift of the radial frequency ω_x. This leads to on site interaction term $U = 2(-1)^{\delta} F \eta_x^4$, where F is the strength of the dipole interaction provided by the standing wave, and $\delta = 0(1)$ corresponds to ions being trapped at the maxima (minima) of the trapping potential. For $F \sim \omega_x$ and $\eta_x^4 = 5.26 \times 10^{-5}$, we find a typical $U = 235$ Hz which leads to $J/U \sim 2$. Note that since $J, U \ll \omega_x$, the number of phonons are conserved. We stress that it is possible to tune J/U to as low as 0.09 by suitably choosing $F \sim 25\omega_x$ with 300 nm standing wave and the radial frequency shifts to $\omega_x \to 0.52\omega_x$ $(1.31\omega_x)$ for repulsive (attractive) interactions; the system still remains in the phonon number conserving regime where $F\eta_x^2 << \omega_x$. The standing wave pattern will be made by on-to-one focussing on each ion which eventually allows individual addressing.

To implement a local quench on the prepared chain of Doppler cooled ions, we prepare a series of steps as shown in Fig. 2. The ion system, after being side band cooled to the motional ground state, is first prepared for simulation [step(a)]. A Raman type PI pulse with one blue sideband and one carrier photon applied to all ions then produces a state with one phonon each at all the sites [step (b)]. It is well-known, that the low-energy Hamiltonian of these phonons can be represented by the Bose-Hubbard model given by [6]

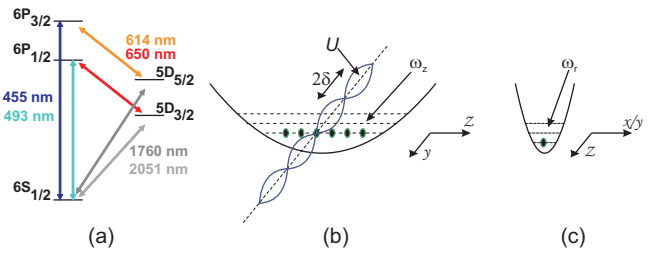

FIG. 1: (Color online) (a) Relevant atomic levels of Ba^+ ions. The Zeeman sublevels are not shown for clarity. (b) Schematic representation of the trapped ion chain. (c) The harmonic oscillator levels in the radial trap.

$$H = J \sum_{\langle ij \rangle} (b_i^\dagger b_j + \text{h.c.}) + U \sum_i \hat{n}_i(\hat{n}_i - 1) + \omega_x \sum_i \hat{n}_i, \quad (1)$$

where b_i denotes the annihilation operator of the bosons (phonons) at site i and $\hat{n}_i = b_i^\dagger b_i$ is the local density operator. Note that Eq. 1 is identical to the Bose-Hubbard Hamiltonian in optical lattices with $U/2 \to U$ [6]. To emulate the effect of the quench of U at a given site, the applied single-site interaction produced by the standing wave is suddenly quenched by changing the phase of the wave by means of a peizo actuator with frequency $\omega_0 \sim 50 - 100$KHz. The frequency ω_0 is chosen such that it keeps the system in the phonon number conserving regime ($\omega_0 \ll \omega_x$) and that it is fast compared to typical energy scales of the underlying Bose-Hubbard Hamiltonian ($\omega_0 \gg J, U$). This leads to the implementation of a local quench protocol. We stress that the ion-trap system provides a large enough window where quench dynamics of its underlying Bose-Hubbard Hamiltonian can be studied accurately within the number conserving approximation. In the rest of this paper, we restrict ourselves to this regime where $\omega_x \gg \omega_0 \gg J, U$.

After the implementation of the local-quench protocol, we aim to measure the subsequent phonon distribution as a function of time. To this end, we note that the time dependence of the phonon distribution is reflected in the time-dependent local population of the phonons $n_{i0}(t)$ at a specified site (i) in a specified motional state, namely the ground state denoted by the index 0 in this case. Thus we aim to measure the time-dependent population of the state $|S_{\frac{1}{2}}, 0\rangle$ after the quench has been performed. In order to measure this population, a red-sideband PI pulse between $S_{\frac{1}{2}}$ and $D_{\frac{5}{2}}$ states are applied so as to

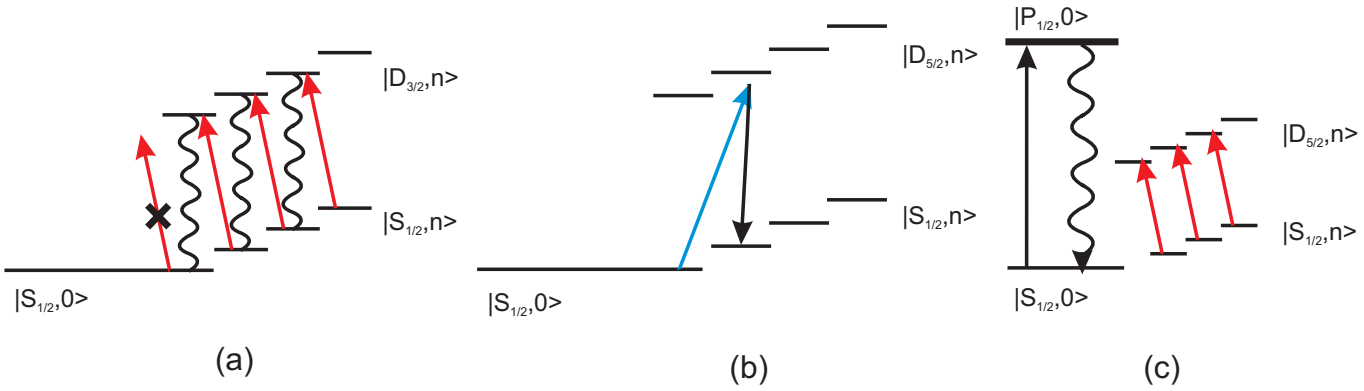

FIG. 2: (Color online) A sequence of laser pulses that are needed to be driven in order to implement site specific quenching. (a) Sideband cooling laser applied continuously in order to prepare the 1D lattice in the ground state of the system. (b) A Raman PI pulse is applied in order to prepare the lattice in an initial near Mott-Insulator state. This pulse is applied to all the sites. (c) This is the final step after the quench pulse has been applied as described in the text. In this step shelving of all other states apart from the motional ground state is performed by a red sideband (RSB) PI pulse.

shelve all other motional state population except that of $|S_{\frac{1}{2}},0\rangle$. Note that the state $|S_{\frac{1}{2}},0\rangle$ is decoupled from this transition. Therefore a $S-P$ fluorescence count will give the population in the motional ground state [step (c)]. The photon count rates can be measured within a time window of 1 second which is much larger than the life time of the excited state $P_{1/2}$ (7.8 ns). The total emitted photon count can be expressed as

$$R_i(t) \;=\; \langle R\rangle n_{i0}(t) = n_{i0}(t) f\omega\Gamma Q_e Q_o/2, \qquad (2)$$

where $\langle R\rangle$ is the mean fluorescence photon count rate; $f = 0.73$ is the branching ratio between $P_{1/2}$ and $S_{1/2}$ state, $\omega = \Omega/4\pi = 1/2[1-\sqrt{1-NA^2}] = 0.04$ (where $NA = 0.4$ is the numerical aperture of a fluorescence collection Halo lens) is the fractional solid angle of detection, Q_e is the quantum efficiency of a photomultiplier tube (PMT) detector which can be estimated to be about 50% at wavelength of 493 nm and $Q_o = 0.1$ is the overall loss factor in collecting the fluorescence photons in the experimental setup [16]. Such a fluorescence count measurement as function of time therefore yields the requisite information on the time-dependent non-equilibrium population distribution of the phonons subsequent to a local quench. In what follows, we shall provide a theoretical estimate of this number for various parameter regime of the Bose-Hubbard model.

To obtain a theoretical understanding of the dynamics of the Bose-Hubbard model when the interaction parameter is quenched from U_i to U_f, we use exact diagonalization to obtain the ground state of the system for $U = U_i$ and all states and energies of the system at $U = U_f$. Since the total phonon number of the system is conserved, we restrict ourselves to those states in the Hilbert space which has a fixed number of total phonons (bosons). In the rest of this paper, we take this total number of phonons to be equal to the number of sites.

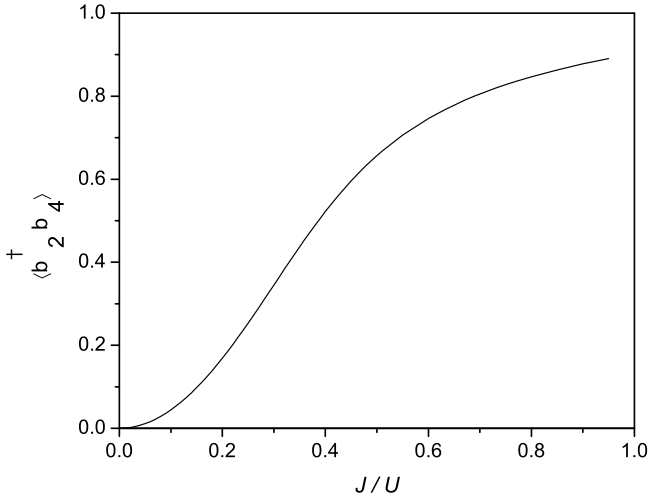

FIG. 3: Correlation function of the bosons as a function of J/U indicating a finite-sized crossover from the Mott to the superfluid phase.

We denote the initial ground state of the model by $|\psi_G\rangle$ and the energy eigenstates and eigenvalues at $U = U_f$ after the quench by $|\alpha\rangle$ and E_α respectively. The wavefunction of the system at a time t after the quench at $t = 0$ can then be found by solving the Schrodinger equation $i\hbar\partial_t|\psi(t)\rangle = H|\psi(t)\rangle$ with the initial condition $|\psi(t=0)\rangle = |\psi_G\rangle$. Expanding the wavefunction $|\psi(t)\rangle = \sum_\alpha c_\alpha(t)|\alpha\rangle$, one finds

$$|\psi(t)\rangle \;=\; \sum_\alpha c_\alpha^0 e^{-iE_\alpha t/\hbar}|\alpha\rangle, \quad c_\alpha^0 = \langle\alpha|\psi_G\rangle. \qquad (3)$$

The time evolution of any operator $\hat{O}_i$, where i is the site index, can be obtained from this wavefunction as $\langle\hat{O}_i\rangle(t) = \sum_{\alpha,\beta} c_\alpha^{0*} c_\beta^0 e^{i(E_\alpha - E_\beta)t/\hbar}\langle\alpha|\hat{O}_i|\beta\rangle$. In the rest

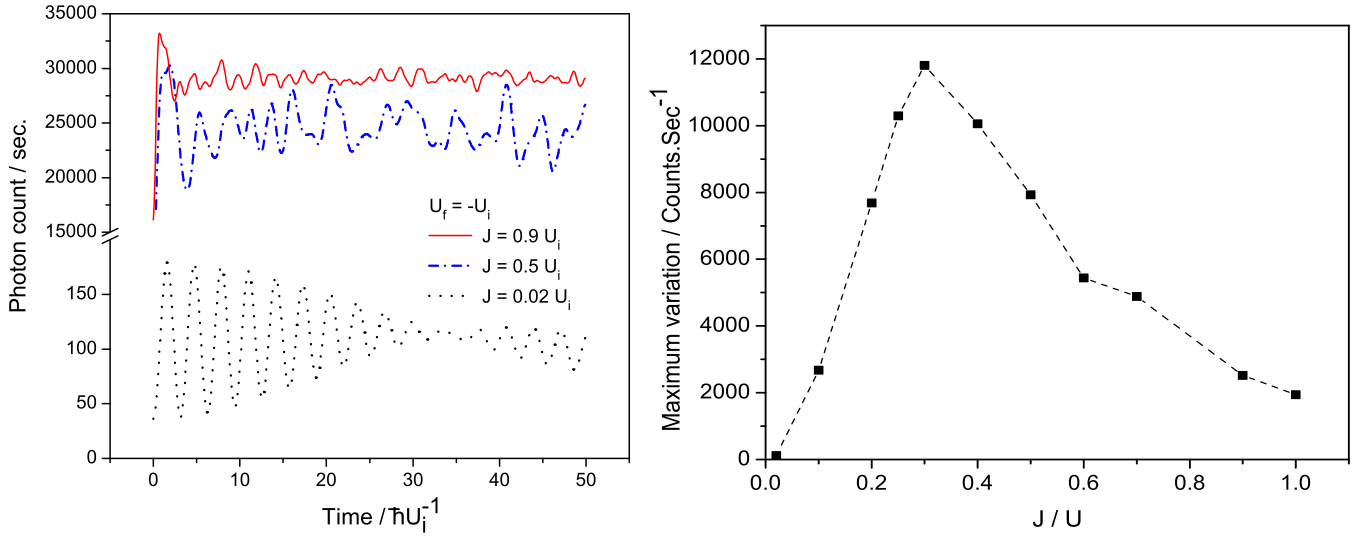

FIG. 4: (Color online) Left panel: Count rate of photons detected per second as a function of time (in units of on-site interaction strength). Right panel: Maximum variation of count rate of photons detected per sec. at different initial values of J/U before the applied quench.

of this work, we shall focus on the time evolution of the projection of the boson number density on the motional ground state of the phonons which corresponds to $\hat{O}_i = \hat{n}_{0i}$ and is given by

$$n_{i0}(t) = \sum_{\alpha,\beta,k_0} c_\alpha^{0*} c_\beta^0 \cos[(E_\alpha - E_\beta)t/\hbar] d^i_{\alpha k_0} d^i_{k_0 \beta}, \quad (4)$$

where $d^i_{\alpha k_0} = \langle \alpha | \Psi^i_{k_0} \rangle$ is the projection of the energy eigenstate $|\alpha\rangle$ onto the state $|\Psi^i_{k_0}\rangle$ which has zero phonons at the i$^{\text{th}}$ site, and the sum over k_0 indicates a sum over all such states in the restricted Hilbert space.

The equilibrium phase diagram of the Hubbard model as a function of J/U, as obtained by exact diagonalization is shown in Fig. 3 where the order parameter correlation $\Delta_{i,j} = \langle b_i^\dagger b_j \rangle$ is plotted as a function of J/U. We find that the plot demonstrates a gradual finite-size crossover from the Mott phase ($\Delta = 0$) at small J/U to a superfluid phase as J/U is increased. To study the effect of local quench, we now prepare the system in its ground states with several different U, switch $U_i \to -U_i$ at the i$^{\text{th}}$ site with the peizo actuator, and measure the photon count $R_i(t)$ which reflects $n_{i0}(t)$ as a function of time (Eq. 2). The resulting photon number fluctuation as a function of time after the applied quench is shown in the left panel of Fig. 4. We find, in accordance with earlier theoretical predictions in Refs. [17, 18], that the amplitude of oscillations of the phonon distribution is maximal when U_i is in the crossover region as shown in the right panel of Fig. 4. Further, we find that the response of the system to the quench is minimal in the Mott phase since n_i approximately commutes with H for small J/U (the commutation is exact for $J = 0$) and hence can not change appreciably as a function of time. From Fig. 4, we estimate that it is feasible to detect a variation of about 30% in the photon count rate (in a count rate of about 24, 000 photons per sec) for different interaction times in the crossover regime [19]. This is well within the present experimental reach.

In conclusion, we have suggested an experimental proposal of detecting time evolution of the phonons in a motional ground state of phonons in an ion trap system subsequent to the *local* change in the effective phonon interaction in one of the ion sites. Our work shows that such an experiment would shed light on the properties of the local dynamics of the Bose-Hubbard model obeyed by these phonons which has not been experimentally studied before. Our supporting theoretical analysis can be easily generalized to initial thermal states of the bosons, to quenches with finite ramp rates, and to other models emulated by these systems.

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